Chapter 35 **Dynamic Decision Theory**

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Abstract This chapter considers the controversial relationship between dynamic choice models, which depict a series of choices over time, and the more familiar atic choice models, which depict a single 'one-shot-only' decision. An initial sue concerns how to reconcile the normative advice of these two models: Should and agent take account of the broader dynamic context when making a decision, and if so, in a sophisticated manner (the orthodox backwards induction approach), a rather in a resolute manner (which takes the past as well as the future to be gnificant)? Further controversies concern what the dynamic implications of an stent's preferences reveal about the (ir)rationality of these preferences.

35.1 'Dynamic' Versus 'Static' Decision Theory

On paper, at least, dynamic (otherwise known as sequential) and static decision models look very different. The static model has familiar tabular or normal form, with each row representing an available act/option, and columns representing the different possible states of the world, yielding different outcomes for each act. Such models apparently depict a single 'one shot only' decision. Dynamic models, on the other hand, have tree or extensive form—they depict a series of anticipated choice points, the later choices often following the resolution of some uncertainty.

These basic differences between the two types of models raise a number of questions about how, in fact, they relate to each other:

- Do dynamic and static decision models depict the same kind of decision
- * If so, what is the static counterpart of a dynamic decision model? Ultimately: How should one initiate a sequence of choices?

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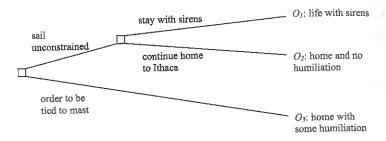


Fig. 35.1 Ulysses' dynamic decision problem

· Do dynamic decision models shed light on old normative questions concerning learning and choice rules?

These issues turn out to be rather controversial; this chapter will consider them in turn. Firstly, however, it is helpful to set the scene with a couple of examples.

A well-known dynamic decision problem is the one facing Ulysses on his journey home to Ithaca in Homer's great tale from antiquity. Ulysses must make a choice about the manner in which he will sail past an island inhabited by sweet-singing sirens, knowing that once he reaches the island, he must then choose whether to stop there indefinitely or to keep sailing. Ulysses' initial choice concerns whether to order the crew to tie him to the mast when nearing the island. If he makes the order, he will later have no further choices and the ship will sail onwards to Ithaca. If he does not make the order, he will later have the choice mentioned above. The final outcome depends on what sequence of choices Ulysses makes. The problem can be depicted in extensive form, as per Fig. 35.1. The square nodes represent the two choice points.

The second problem will be described only in abstract form: It is given in Fig. 35.2.1 The model illustrates a dynamic decision involving some uncertainty. (This particular problem will also be useful for our discussion in later sections.) Circle nodes indicate points of uncertainty, where all of the branches have some probability of occurring, as per the beliefs of the agent in question. Square nodes represent choice points, as before, where the branches are the options the agent perceives as available at that choice point. In this particular decision problem, the first uncertainty to be resolved concerns whether some event E, or else its complement $\sim E$ turns out to be the case. The later uncertainty concerns whether event F or $\sim F$ is true. O_1 , O_2 , and O_3 refer to possible outcomes of the agent's sequence of choices, and delta is some small positive amount such that, say, $O_3 - \delta$ is slightly less preferable than O_3 .

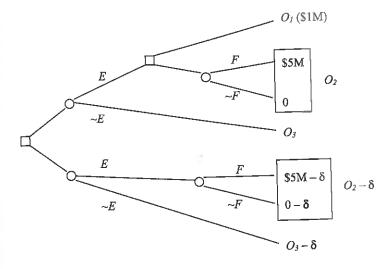


Fig. 35.2 Dynamic decision problem with uncertainty

Was Ulysses Rational?

It has been argued (e.g., [20], 202) that Ulysses' plight, while interesting from a dynamic perspective, is not in fact the appropriate topic for dynamic decision theory, because Ulysses is a flawed agent: he expects a change in attitude towards the sirens, for no good reason. According to Homer's story, Ulysses suffers a kind of weakness of will when he hears the sirens singing, and spontaneously changes his preferences, despite there being no new information available. This is an imperfect agent with problematic belief and desire changes, and not the appropriate subject for dynamic decision theory, or so the argument might go.

There is something to this line, but it introduces an uncomfortable rift: Dynamic and static decision theory must then deal with different subject matter. Ulysses is deemed irrational from the dynamic perspective, and thus not worthy of attention. On the other hand, surely decision theory has something to offer agents facing predicaments like Ulysses'. This is presumably the job of static decision theory.

One may certainly approach the relationship between dynamic and static decision theory in this way, namely that the former is the yardstick for assessing an agent like Ulysses' rationality over some period of time, perhaps his whole lifetime, while static decision theory assesses his rationality only at the 'present' time. In this way, Ulysses may be rational in the static sense when he orders the crew to tie him to the mast, while falling short of rationality in the more demanding, dynamic sense.

The worry is that the latter demanding notion of rationality—one that concerns an agent over an extended period of time—is of mere intellectual interest. If the aim is to offer pertinent practical guidance to an agent, it seems more fruitful to regard dynamic and static decision theory as concerned with the same decision problem,

¹This decision problem is from Rabinowicz [21, 599].

namely: What constitutes rational choice at the 'present' live point of decision? The dynamic model can be understood simply as a nice way of visualising the temporal series of choices and learning events that an agent predicts she will confront. On this reading, the key question, then, is: How should an agent choose her initial move in light of her predicted decision tree? To answer this, we need to determine how the future nodes of the decision tree should bear on the initial choice. How should the agent conceive, in static terms, of her choice problem?

35.3 How Should One Initiate a Sequence of Choices?

The questions left dangling in the previous section have generated a surprising amount of controversy. Three major approaches to dynamic choice have appeared in the literature. These are the *naive* or *myopic* approach, the *resolute* approach and the *sophisticated* approach. I join a number of others (e.g. [16, 18, 23]) in defending the latter, but there are also steadfast supporters of resolute choice (notably [20] and [17]). Myopic choice is best conceived as a useful contrast for the other two approaches.

Let us begin with the contrast case. A naive agent assumes that any path through the decision tree is possible, and so sets off in pursuit of whichever path they calculate to be optimal, given their present attitudes. For instance, a naive Ulysses would simply presume that he has three overall strategies to choose from: Either ordering the crew to tie him to the mast, or issuing no such order and later stopping at the sirens' island, or issuing no such order and later sticking his course. Ulysses prefers the outcome associated with the latter combination, and so he initiates this strategy by not ordering the crew to restrain him. Table 35.1 presents naive Ulysses' static decision problem. In effect, this decision model does not take into account Ulysses' present predictions regarding his future preferences.

There is no need to labour the point that the naive approach to dynamic choice is aptly named. Ulysses chooses to 'sail unconstrained and then go home to Ithaca', but, by his own lights, this combination of choices would not be realised; initiating the act would inevitably lead Ulysses to stay on the island of the sirens. The hallmark of the sophisticated approach, by contrast, is its emphasis on backwards planning: The sophisticated chooser does not assume that all paths through the decision tree, or in other words, all possible combinations of choices at the various choice nodes, are genuine options. The agent considers, rather, what they would be inclined to choose at later choice nodes if they were to arrive at the node in question. Indeed, the agent starts with the final choice nodes in the tree, and considers what would be

Table 35.1 Naive Ulysses

Act	Outcome
0.4	Life with sirens
Sail unconstrained then home to Ithaca	Reach home, no humiliation
Order tying to mast	Reach home, some humiliation

Table 35.2 Sophisticated Ulysses I

Act	Outcome
Sail unconstrained then stay with sirens	Life with sirens
	Reach home, some humiliation

Table 35.3 Sophisticated Ulysses II

Act	Later prefer sirens $(p = 1)$	Later prefer Ithaca $(p=0)$
Sail unconstrained		Home, no humiliation
Order tying to mast	Home, some humiliation	Home, some humiliation

chosen at these nodes, given their predicted preferences at each of these positions. These predicted choices would presumably affect what the agent would choose at the second-last choice nodes. Once the second-last choices have been determined, the agent moves to the third-last choice nodes, and so on back to the initial choice node. The result is that only certain paths through the decision tree would ever be realised, if initiated.

Sophisticated Ulysses would take note of the fact that, if he reaches the island of the sirens unrestrained, he will want to stop there indefinitely, due to the transformative effect of the sirens' song on his preferences. He acknowledges the implications of this prediction and so deems the choice combination of 'not issuing an order for his crew to restrain him and then sticking to his course' to be an impossibility. Table 35.2 presents sophisticated Ulysses' static representation of his decision problem in terms of the combinations of choices that he predicts would arise. Note that there are only two feasible combinations of choices here. Table 35.3 gives an alternative static representation, whereby Ulysses' future preferences/choices are part of the state space; this static representation is, in a sense, more general, because it can easily be modified to incorporate probabilistic, as opposed to certain, predictions about future preferences/choices, (In Table 35.3, the later preference for staying with the sirens is certain; we see that the probability for this state, p, is equal to one.) Moreover, in Table 35.3, the 'acts' are limited to those things the agent can initiate at the moment of decision. This is in keeping with, for instance, Joyce's [11, 57-61] interpretation of acts in a static decision model.

Defenders of the resolute approach to dynamic choice dismiss problems like Ulysses', claiming that Ulysses cannot serve as a model agent for dynamic rationality, for the reason given in Sect. 35.2. Indeed, the resolute approach is particularly unconvincing in the context of Ulysses' decision problem. The key point of difference between the sophisticated and resolute approaches concerns how a rational agent may be expected to choose at future nodes of a decision tree. While the sophisticated approach assumes that an agent always chooses in accordance with their preferences at the time, the resolute approach holds that an agent may sometimes defer to their previous preferences or strategy—they may honour a previous commitment, despite present misgivings. The reason this recommendation is not very convincing in Ulysses' case is that there is no apparent reason why Ulysses, upon reaching the island of the sirens and not restrained by his crew,

would ignore his current preferences and instead sail straight on. Indeed, depending how one understands the relationship between preference and choice, it is arguably contradictory to depict an agent choosing according to preferences other than their own preferences at the time.

Agents like Ulysses cannot appeal to the resolute approach to dynamic choice. Defenders of the resolute approach rather appeal to decision problems like the one in Fig. 35.2. Their aim is to defend both the resolute approach to dynamic choice and preferences that violate the *independence* axiom.² The agent's preferences, with respect to Fig. 35.2, are thus stipulated as follows (so as to violate independence): at all times she prefers O_1 to O_2 , but she prefers the lottery that gives O_2 if E and E and E and E to the lottery that gives E and E and E and the latter as E and the latte

Some examination reveals that a sophisticated agent with preferences as specified above (or more accurately, who *predicts* that her preferences will be as specified above) initially chooses 'down' in the problem in Fig. 35.2, which effectively amounts to the lottery $L_2 - \delta$. Note that this strategy is dominated by the one that amounts to L_2 : 'up' at the initial choice node, and then 'down' at the second. According to McClennen [20], this is precisely the kind of situation in which the resolute, as opposed to the sophisticated, approach to the problem is more apt. At all times the agent prefers L_2 to $L_2 - \delta$, so it is in her best all-round interests to pursue the L_2 lottery and stick firmly to this plan, despite the fact that O_I will look better than O_2 at the second choice node (i.e., 'up' rather than 'down'), should the agent reach this position.

We need not get sidetracked here by questions about the (ir)rationality of preferences that violate independence. The agent's preferences can simply be taken as given. The question is: Can such an agent reasonably expect to be a resolute chooser? That is: Would an agent with preferences as stipulated reasonably choose 'down' rather than 'up', were she to reach the second choice node in Fig. 35.2? Defenders of resolute choice say that the rational agent would indeed vindicate her previous decision to pursue the lottery L_2 . In this author's opinion, that proposal defies the very meaning of preference. Of course, an agent may place considerable importance on honouring previous commitments. Any such integrity concerns, however, should be reflected in the description of final outcomes and thus in the agent's actual preferences at the time in question. Conceiving an agent's preferences as concerning more complex outcomes than initially supposed, is quite different from conceiving an agent's preferences to be out of step with her supposed choices at the time in question, which is what the resolute approach to sequential choice is committed to.

What this discussion highlights is that controversies surrounding dynamic/sequential choice are essentially controversies about how an agent's decision at a time should be informed by her predicted future preferences/choices (in addition to predictions about future states of affairs). The naive approach recommends that an agent simply ignore predictions about her future attitudes. This is clearly problematic as it amounts to not taking into account all the available evidence when making a decision. The dispute between the resolute and sophisticated approaches is more fine-grained; it concerns how predicted future preferences inform corresponding future choices, in the context of the greater dynamic decision problem at hand.

While not orthodox, the most general translation of a dynamic decision problem to static form is to include future preferences/choices in the state space (see [29], Sect. 35.2). Table 35.3 employs this representation. The available acts are simply the options at the initial choice node. The outcomes of these acts depend not only on how things turn out in the external world, but also on what decisions the agent confronts later and the strategies she would then choose. These may all be aspects of the future that the agent is unsure about, and thus assigns probabilities between zero and one. (Note that the examples in this chapter, as per much of the discussion of sequential choice, are special cases where future preferences are known for sure, or in other words, as in Table 35.3, are assigned probability one.)

35.4 Normative Questions: Can Dynamic Decision Models Help?

We have seen how dynamic decision trees can help an agent like Ulysses take stock of his static decision problem, so that he can work out what to do 'now'. The literature on dynamic decision theory has more ambitious aims than this, however. Much discussion is devoted to more general normative questions: Must rational preferences conform to expected utility theory? What constitutes rational belief and preference change? Indeed, the work of a number of authors, including Hammond [6–10], Seidenfeld [23–27], McClennen [19, 20], Machina [17], Rabinowicz [21, 22], Skyrms [28], Steele [29], Bradley and Steele [3], and Buchak [4, 5] demonstrates that dynamic decision models provide a rich setting for investigating these familiar normative questions. As one might guess, the findings are controversial.

Our earlier discussion of the resolute approach to dynamic choice gave a glimpse of how dynamic-choice problems shed light on normative issues. Refer back to the problem in Fig. 35.2. This problem is useful for evaluating preferences that violate independence (as per cumulative prospect theory (see [17]) and the associated riskweighted expected utility theory defended by Buchak [5]). The preferences specified in Sect. 35.3 violate independence, by design:

²Joyce [11, 86] gives the following informal statement of the *independence* axiom: 'a rational agent's preferences between (acts) A and A* should not depend on circumstances where the two yield identical outcomes.'

 $O_1 > O_2$ $L_2 : (O_2 \text{ if } E; O_3 \text{ if } \neg E) > L_2 - \delta > L_1 : (O_1 \text{ if } E; O_3 \text{ if } \neg E)$

The dynamic choice problem in Fig. 35.2 can serve as a controlled experiment, so to speak, to test preferences of this sort. The experiment is 'controlled' because the agent has stable or constant preferences; she does not predict any rogue changes in belief or desire, as per Ulysses. Indeed, the agent predicts her preferences will change only due to learning new information that leads to a belief update in accordance with Bayes' rule. The question is whether these preferences are shown to be in some sense self-defeating, suggesting they are irrational.

Recall that the sophisticated agent with preferences as specified above will choose 'down' in Fig. 35.2, effectively selecting a strategy that amounts to $L_2 - \delta$. The embarrassment here is that there is another strategy in the dynamic tree—'up' initially and then 'down'—that effectively amounts to L_2 , which clearly dominates $L_2 - \delta$. Of course, our agent is not guilty of choosing an option that is dominated by another available option. The problem, however, is that it is the agent's own preferences that make the dominating L_2 strategy unavailable to her. And for this reason, we might judge the preferences to be self-defeating or irrational.

A number of the authors mentioned above discuss this potential criterion for rational preferences, namely that dominating strategies in the dynamic setting should not be unavailable to an agent on account of her own (stable) preferences. Refer to this as the 'dominating-strategies' criterion. McClennen [20] upholds the criterion, and Rabinowicz [21] and Steele [29] express some support for it. Hammond [6, 7, 9, 10] defends an even stronger criterion that effectively requires all strategies in an extensive-form model, i.e., all combinations of choices, to be available to an agent. In other words, the agent's own preferences should not prevent her from pursuing what she 'now', i.e., at the outset, considers to be the best strategy. Hammond refers to this criterion as *consequentialism*, but this label is rather misleading.

Assuming sophisticated (rather than resolute) choice, the 'dominating-strategies' criterion rules out preferences that violate independence.³ It is worth noting that this same criterion is the cornerstone of the well-known 'diachronic Dutch book argument', or at least the version in Skyrms [28]. In this case, the 'controlled experiment' takes a slightly different form: The agent at all times has preferences that conform to expected utility theory, and her basic desires are stable. It is the agent's learning or belief-update rule that is under scrutiny. Skyrms shows that a sophisticated agent whose belief-update rule is something other than *Bayesian conditioning*, may, in some cases, choose a dominated option because the dominating option is unavailable to her. Conversely, this is never the case for an agent who plans to update by Bayes' rule. On this basis, we are supposed to conclude that Bayesian conditioning is the uniquely rational belief-update rule.

Seidenfeld notably rejects both Hammond's consequentialism and the 'dominating-strategies' criterion just outlined. Seidenfeld seeks to defend decision theories that violate ordering (and only secondarily, independence); an example of such a theory is Levi's [15] *E-admissability* choice rule. Like cumulative prospect theory (for instance), Levi's theory, in its general form, does not satisfy the 'dominating strategies' criterion (see [29]). Unlike cumulative prospect theory, however, Levi's theory does satisfy an alternative criterion for rational preference that concerns future choices between 'indifferents', or options the agent is indifferent between; Seidenfeld articulates and defends this criterion in a series of interchanges with other authors (i.e., in [23, 24, 26, 27]); the debate is discussed in detail in Steele [29].

We might label Seidenfeld's criterion the 'future indifferents' criterion. It holds that, in the controlled setting where preferences remain stable, if the agent will be indifferent between options at a later choice node, then she should be indifferent now between strategies that terminate in these options, and are otherwise identical prior to the choice node in question. If this criterion is not satisfied, as per theories that violate independence like cumulative prospect theory [23], there is no 'natural' way to evaluate the aforementioned strategies. The strategies have differing utilities, but either one may eventuate if the agent makes the appropriate initial choices. The obvious move here is to acknowledge only one strategy, with a final tie-breaking step. Steele [29] pursues this line of argument against Seidenfeld's criterion. There remain problems, however, if the evaluation of the single strategy depends on which tie-breaker is selected, as per preferences that violate independence.

Whether or not one affirms Seidenfeld's 'future indifferents' criterion for rational preference, the issues it raises demand consideration. Choice in the face of indifference has always been puzzling, but the problems take on new significance in the dynamic setting—in this setting there is a need to explicitly model predicted future choices, including choices between indifferents, in order to evaluate current options.

35.5 Concluding Remarks

The previous section gave a tour of the prominent dynamic-choice arguments concerning rational preference (and learning) that have been discussed in the literature. As indicated, there is persistent disagreement. Some of the disagreement concerns the sophisticated/resolute distinction discussed in Sect. 35.3. This is best considered a dispute about the meaning of the terms in a dynamic-choice model, in

³We thus see why McClennan [20] defends the resolute approach to dynamic choice. For similar reasons, Buchak [5] is also sympathetic to resolute choice.

⁴This is a lexical choice rule that can handle indeterminate belief and/or desire, represented by a set of probability-utility function pairs. The 'E-admissible' options are those that have maximal expected utility for at least one probability-utility representation in the set; these are the options that a rational agent *may permissibly* choose. The agent discriminates between 'E-admissible' options on the basis of her 'security' attitudes.

particular, future preference and its relationship to future choice. Beyond that, there is disagreement about what are reasonable features of choice in the dynamic setting as discussed in Sect. 35.4.

A further topic of debate in the more recent literature concerns what rival decision theories say about the value of 'information' or evidence retrieval. This implicant dynamic choice as it concerns whether to choose one of a given set of options 'now or rather wait to collect evidence that may be pertinent to the choice in question A candidate rationality criterion is that one should always wait to retrieve further evidence if the evidence is 'free' and may influence one's choice; call this the 'free evidence' criterion. The criterion is discussed in Kadane et al. [12], Buchal [4], and Bradley and Steele [3] in relation to varying generalisations of expected utility theory. These authors reject the standard version of the 'free evidence' criterion, but arguably, the issues are not fully settled. Another topic deserving of further investigation is the possibilities for rational preference change; Bradle [2], for instance, considers cases beyond preference change in response to new information. The relationship between present uncertainty about future preferences and a 'preference for flexibility' with respect to available options in the future is a related issue that also deserves further investigation; for early works on this topic. see Koopmans [13], Kreps and Porteus [14], and Arrow [1].

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Asterisks (*) indicate recommended readings.

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